DAMTP R-90/20

THE EFFECT OF BOUNDARIES IN ONE-LOOP QUANTUM COSMOLOGY*

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September 1990

Abstract. The problem of boundary conditions in a supersymmetric theory of quantum cosmology is studied, with application to the one-loop prefactor in the quantum amplitude. Our background cosmological model is flat Euclidean space bounded by a three-sphere, and our calculations are based on the generalized Riemann ζ-function. One possible set of supersymmetric local boundary conditions involves field strengths for spins 1, $\frac{3}{2}$ and 2, the undifferentiated spin-$\frac{1}{2}$ field, and a mixture of Dirichlet and Neumann conditions for spin 0. In this case the results we can obtain are : $\zeta(0) = \frac{7}{45}$ for a complex scalar field, $\zeta(0) = \frac{11}{360}$ for spin $\frac{1}{2}$, $\zeta(0) = -\frac{77}{180}$ (magnetic) and $\frac{13}{180}$ (electric) for spin 1, and $\zeta(0) = \frac{112}{45}$ for pure gravity when the linearized magnetic curvature is vanishing on S^3. The $\zeta(0)$ values for gauge fields have been obtained by working only with physical degrees

of freedom. An alternative set of boundary conditions can be motivated by studying transformation properties under local supersymmetry; these involve Dirichlet conditions for the spin-2 and spin-1 fields, a mixture of Dirichlet and Neumann conditions for spin-0, and local boundary conditions for the spin-$\frac{1}{2}$ field and the spin-$\frac{3}{2}$ potential. For the latter one finds : $\zeta(0) = -\frac{289}{360}$. The full $\zeta(0)$ does not vanish in extended supergravity theories, indicating that supersymmetry is one-loop divergent in the presence of boundaries.

*Published in Proceedings of the IX Italian National Congress of General Relativity and Gravitational Physics, Capri, 25-28 September 1990 (World Scientific Publishing Co.).

This research is concerned with the problem of the one-loop finiteness of amplitudes in quantum cosmology in the presence of boundaries. The calculations are performed using the generalized Riemann ζ-function, working in a particular background with specified boundary conditions. The value $\zeta(0)$ at the origin provides information about one-loop divergences of the quantum amplitudes with the prescribed boundary conditions. One then has to check whether the contributions to $\zeta(0)$ from bosonic and fermionic fields add up to zero for a suitable supergravity model.

We consider a background cosmological model given by flat Euclidean space bounded by a three-sphere (hereafter referred to as S^3). For fermionic fields one has a choice of local and non-local boundary conditions because of the first-order nature of the Dirac operator. The former are of greater interest because motivated by supersymmetry. Using two-component spinor notation, one possible set of local boundary conditions, involving field strengths for a spin-s field and the normal to S^3 is [1] :

$$2^s \, n^{AA'} ... n^{LL'} \phi_{A...L} = \pm \widetilde{\phi}^{A'...L'} \quad . \tag{1}$$

In (1), $n^{AA'} = n^a \sigma_a^{AA'}$ is the spinor version of the normal to S^3, $\phi_{A...L}$ and $\widetilde{\phi}_{A'...L'}$ are field strengths not related by complex conjugation. For a complex scalar field, the real part obeys a Dirichlet (D) and the imaginary part a Neumann (N) condition (or viceversa). Supersymmetry plays a role in (1) because in the case of a flat Euclidean background bounded by S^3 there is a spin-lowering operator for solutions to the linearized massless free-field equations which preserves these local boundary conditions [2]. This generates rigid supersymmetry transformations among classical solutions obeying (1) on S^3. However, these rigid transformations do not map *eigenfunctions* of the spin-s wave operators to eigenfunctions for adjacent spin $s \pm \frac{1}{2}$ with the *same* eigenvalues. Thus there is no *a priori* reason for $\zeta(0)$ values for adjacent spins to be equal; the $\zeta(0)$ value for each spin must be calculated separately. For bosonic fields, conditions (1) imply the vanishing on S^3 of the magnetic (B) or electric (E) field for spin 1, whereas for pure gravity (PG) the linearized magnetic curvature is vanishing on S^3 (the fixing of the linearized electric curvature would lead to an ill-posed classical boundary-value problem). For all gauge fields we shall be here concerned with the method of reduction of the theory to its physical degrees of freedom (hereafter referred to as PDF), so as to complete previous work appearing in the literature for bosonic fields [3]. Results obtained using a quantization technique based on the Faddeev-Popov method can be found in [4,5]. The PDF results for bosonic fields (compare with [4,5]) are : $\zeta_B(0) = -\frac{77}{180}$, $\zeta_E(0) = \frac{13}{180}$, $\zeta_{PG}(0) = \frac{112}{45}$, and for a complex scalar field one finds : $\zeta(0) = \zeta_D(0) + \zeta_N(0) = -\frac{1}{180} + \frac{29}{180} = \frac{7}{45}$. For the spin-$\frac{1}{2}$ field, we have found that a first-order differential operator for the local boundary-value problem (1) exists which is symmetric and has self-adjoint extensions [2,6]. A *direct* calculation [2,6] gives :

$$\zeta_{\frac{1}{2}}(0) = \frac{11}{360} \quad . \tag{2}$$

For the spin-$\frac{3}{2}$ field, it is not yet clear whether, and eventually how, conditions (1) can be used to perform one-loop calculations.

An alternative set of local boundary conditions [7] is suggested by the study of field transformation properties under local supersymmetry. These involve the spatial components $\left(\psi_i^A, \ \widetilde{\psi}_i^{A'}\right)$ of the spin-$\frac{3}{2}$ potential, rather than the field strength $\left(\phi_{ABC}, \ \widetilde{\phi}_{A'B'C'}\right)$.

In particular, in simple supergravity the spatial tetrad and the projection $\Bigr(\pm \widetilde{\psi}_i^{A'} - \sqrt{2}\, n_A^{\ A'} \psi_i^A \Bigr)$ transform into each other under half of the local supersymmetry transformations at the boundary, so that they can be specified as boundary data (up to gauge) in computing the quantum amplitude [7]. Thus from this point of view the most natural boundary conditions are Dirichlet for spin 2, and : $\sqrt{2}\, n_A^{\ A'} \psi_i^A = \pm \widetilde{\psi}_i^{A'}$ on S^3. The PDF value for gravity is $\zeta(0) = -\frac{278}{45}$ in this case [3], and for spin $\frac{3}{2}$ a *direct* $\zeta(0)$ calculation using the above local boundary condition gives [2,6] :

$$\zeta_{\frac{3}{2}}(0) = -\frac{289}{360} \quad , \tag{3}$$

where $\zeta_{\frac{3}{2}}(0)$ is the PDF value, found using the gauge condition $e_{AA'}^{\ \ \ \ j} \psi_j^A = 0$. In the case of $O(N)$ supergravity models, the remaining boundary conditions consistent with supersymmetry transformation rules are Dirichlet for scalar fields (and Neumann for pseudo-scalars), magnetic for spin 1, and the previous local boundary conditions (1) for massless spin-$\frac{1}{2}$ fields. One then finds [2] for the different $O(N)$ models with scale-invariant measures :

$$\zeta_T^{(1)}(0) = -\frac{43}{8},\ \zeta_T^{(2)}(0) = -5,\ \zeta_T^{(3)}(0) = -\frac{61}{12},\ \zeta_T^{(4)}(0) = -\frac{17}{3}, \tag{4}$$

$$\zeta_T^{(5)}(0) = -\frac{41}{6},\ \zeta_T^{(6)}(0) = -\frac{55}{6},\ \zeta_T^{(7)}(0) = \zeta_T^{(8)}(0) = -\frac{83}{6}. \tag{5}$$

Thus $O(N)$ supergravity theories are not even one-loop finite in the presence of boundaries, at least when the PDF values are used for gauge fields. The same conclusion has been reached in [5] using the Faddeev-Popov technique and including the effect of antisymmetric tensor fields, which have not been considered in deriving (4,5).

For fermionic fields, as we said at the beginning, one has an additional choice of using non-local (spectral) boundary conditions, rather than the previous local conditions. This choice is not useful in studying $O(N)$ supergravity models because it does not respect supersymmetry, but it is nevertheless of some mathematical interest. For a massless spin-$\frac{1}{2}$ field, non-local boundary conditions require that the part of the spin-$\frac{1}{2}$ field with a particular sign of eigenvalues of the intrinsic 3-dimensional Dirac operator should vanish on S^3, and similarly for the spin-$\frac{3}{2}$ potential. This leads to [2] :

$$\zeta_{\frac{1}{2}}(0) = \frac{11}{360} \quad , \quad \zeta_{\frac{3}{2}}(0) = -\frac{289}{360} \quad , \tag{6}$$

where, again, $\zeta_{\frac{3}{2}}(0)$ is the PDF value. Remarkably, these are equal to the local values (2,3).

A further topic for investigation is the relation between our PDF $\zeta(0)$ values for gauge fields and the results of BRST-invariant $\zeta(0)$ calculations including ghost fields.

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